Preferences, Utility and the Basics of Simultaneous-Move Games

Economics 222 - Introduction to Game Theory

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Topics

1. Utility and preferences
2. Basic setup for simultaneous-move games

Example

- Suppose we want to study the interaction between a student, Bob, and a professor.
- Bob may go to class or skip class, while the professor may give a pop quiz or not.
- There are therefore four possible outcomes:
  - Bob goes to class, professor gives a pop quiz.
  - Bob skips class, professor gives a pop quiz.
  - Bob goes to class, no pop quiz.
  - Bob skips class, no pop quiz.
- What do we need to know in order to meaningfully analyze the situation?

Preference Relation

- **Binary relation** (mathematical relation between two objects) that characterizes an economic agent's preferences over **alternatives/outcomes**:
  - $x \succsim y$: the agent **prefers** x to y;
  - when both $x \succsim y$ and $y \succsim x$, we can write $x \sim y$: the agent is **indifferent** between x and y;
  - when $x \succsim y$ and NOT $y \succsim x$, we can write $x \succ y$: the agent **strictly prefers** x to y - this means that the agent will not choose y if x is available.
- (We will use the words "alternative" and "outcome" interchangeably.)
- An outcome can be anything: a consumption bundle, an amount of money, a quantity of work, the identity of a love partner, etc.
- An outcome can also be a combination of many things.
- What type of outcomes we consider will depend on the situation at hand.

Describing Preference Relations Can Be Time-Consuming

- In our example, there are 4 outcomes, so there are $\frac{4\times 3}{2} = 6$ pairs to compare. That's not too bad.
- But if there were 15 outcomes, there would be $\frac{15\times 14}{2} = 105$ pairs to compare. Describing such a preference relation would be quite cumbersome!
- How can we describe a preference relation without enumerating each pair of outcomes?

Representation of Preferences by Utility Function

- We say that utility function u **represents** preference relation $\succsim$ when, for any outcomes x and y,

$$u(x) \geq u(y) \text{ if and only if } x \succsim y.$$

- That is: u and $\succsim$ correspond to the exact same ranking of outcomes (including any ties).

What Preferences Can Be Represented by a Utility Function?

- If preference relation $\succsim$ can be represented by a utility function, then $\succsim$ must be **complete** and **transitive**:
  1. Completeness: any two outcomes can be compared by $\succsim$. That is, given any two outcomes x and y, we have $x \succ y$, $x \sim y$, or $y \succ x$.
  2. Transitivity: if $x \succsim y$ and $y \succsim z$, then $x \succsim z$.
- Why?
- Completeness and transitivity are sufficient if the number of outcomes is finite.

Non-Uniqueness of Utility Function

- If u_i represents i's preferences, then any function v that orders all outcomes in exactly the same way as u_i will also represent i's preferences over those outcomes.
- There are infinitely many such functions v.
- Example: If u_1 represents $\succsim$ and u_2 represents $\succsim'$, what can you say about $2u_1 + 3$ and $0.00001u_2$?
- Does it make sense to add or compare utilities of different people?

What is a Game?

- A game has **players** that each chooses from **actions** available to him/her.
- *Example:* Bob and the professor are players. Bob's actions are "Go to Class" and "Skip Class"; the professor's actions are "Give Quiz" and "No Quiz."
- For now, we study simultaneous-move games.
- **Simultaneous-move** means that each player acts once, and does so without knowing others' actions/strategies.
- *Example:* The game would not be simultaneous-move if the professor decides whether to give a quiz after seeing if Bob is in class.

Outcomes and Payoffs

- In a simultaneous-move game, an **outcome** (or action profile) is a collection of actions containing exactly one of each player's actions.
- *Example:* (Go to class, Give quiz) is an outcome.
- Each outcome generates a utility, or **payoff**, for each player.
- Each player's utility function represents her preferences.
    - Therefore, we are implicitly assuming completeness and transitivity.
- (We will need to be more careful about utilities when we introduce uncertainty in the second half of the semester. But for now, we're fine.)

Complete Information

- **Complete information** means that each player knows *every* player's payoff from each outcome.
- *Example:* If lazy students' payoffs differ from hard-working ones', and the professor does not know whether Bob is lazy, then the game would not feature complete information.
- We will assume complete information **except** when we study Bayesian Nash equilibrium (at the end of the course, if time permits).

The Normal Form

- A convenient way to represent a two-player simultaneous move game of complete information is through the **normal form** (also known as strategic form).
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		Prof.	
		Give Quiz	No Quiz
Bob	Go to class	5,3	7,10
	Skip class	0,0	10,6

- By convention, player 1 (Bob) picks the row, and player 2 (professor) picks the column.
- Each cell gives the payoffs of player 1 and player 2, in that order.

What is a Strategy (in a Simultaneous-Move Game)?

- Each player plays a **strategy**, which is, in a simultaneous-move game, a probability distribution over her actions.
- *Example:* "Go to class" is one of Bob's strategies. "Go to class with probability 0.3 and Skip class with probability 0.7" is another.
- A **pure** strategy is a strategy that puts probability 1 on a single action.
- **For the first half of the semester, we will only study pure strategies.**
- A collection of strategies containing exactly one of each player's strategies is called a **strategy profile**.
- *Example:* (Go to class, Give quiz) is a (pure) strategy profile.

Pareto Efficiency

- When is an outcome unambiguously better than another?
- One possible answer: when somebody is better off, and nobody is worse off.
- An outcome is **Pareto dominated** if there exists another outcome where someone is better off, and no one is worse off.
- Mathematically: Suppose the utilities of agents 1 through n are $(u_1, u_2, ..., u_n)$ under outcome a. Then outcome a is Pareto dominated if there exists another outcome b with payoffs $(u'_1, u'_2, ..., u'_n)$, where $u'_i \geq u_i$ for **all** i, **and** $u'_j > u_j$ for **some** j.
  - We then say that b is a **Pareto improvement** over a.
- An outcome is **Pareto efficient** if it is not Pareto dominated.
- Which outcome(s) is/are Pareto efficient in the Bob v. Professor example?